# Information fusion applied to the tracking of GPS pilot and data channels

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**Abstract** – *New GNSS signals broadcast at different carrier frequencies include, most of them, a pilot and data channels. In this article we propose to fuse the information of code and phase delay provided by the two channels in order to improve the position accuracy and to extract the message of navigation. The proposed system is composed of a change point estimate of the slope of discriminator variations and of a circular recursive filter for the estimation and fusion of the carrier phase delay. The contribution of this work is the definition of a recursive change point estimate that fuse the discriminator values and integrate prior information for the detection. The proposed architecture that uses a circular filter enables to fuse the phase delays and to extract the message of navigation. The method is assessed on synthetic data in the experimentation.*

**Keywords:** Tracking, GNSS signals, change point estimation, circular estimation.

## 1 Introduction

A GPS receiver can provide a position every millisecond at frequencies $f_{L1}$ and $f_{L5}$ or every 20 $ms$ at frequency $f_{L2}$ [12, 6]. Nowaday the GNSS systems broadcast at these different frequencies a mix of pilots (data less channels) and data channels. For example a $L2C$ signal is broadcasted at frequency $f_{L2}$ with a time multiplexing of pilot and data channels. For $L5$ pilot and data channels are transmitted with a modulation QPSK (Quadrature Phase Shift Keying). A GNSS receiver must track the parameters of phase and frequency of the carrier and the CDMA (Code Division Multiple Access) code delay. This processing is respectively done by a PLL (Phase-locked loop) and a DLL (Delay-locked loop) systems. In this paper we propose a tracking system that fuses the information of phase and code delay provided by pilot and data channels. The goal of our work is to improve the accuracy of the positioning calculated every millisecond.

In the static case the GPS receiver is at a fix position and its localiqsation processed by the receiver is averaged in time. In this case better positioning is obtained for a longer integration time. In the dynamic case, integration times are smaller in order to take into account the receiver trajectory. So we will have in the navigation case an accuracy that will depend on the receiver speed. In this case there is a trade off between the precision we can obtain and the speed of the receiver. To improve the navigation accuracy the GPS receiver is coupled with dead reckoning sensors. These sensors provide information used to integrate GPS signal, most of the time in an extended Kalman filter. Unfortunately these sensors are in general inaccurate and drift with time, so they can not be used for a long time integration. They are principally used for their robustness because they can provide a position when the GPS receiver fails [5].

In our approach the phase and code delay estimate every millisecond have a low SNR (Signal to Noise Ratio). Then we realise the coherent integration of the signal on several hundreds of milliseconds in order to obtain more precise estimate. The process that provides an estimate at each millisecond is in this case off line. Most of the published works about GPS signal integration concern the problem of localisation for low signal to noise ratios. An application in this case is for example the navigation in cover environments like indoor localisation. The long coherent integration is then realized on several dozen of milliseconds for which the message of navigation is known and the GPS signal supposed to be stationary. In these works an extended Kalman filter is used for the on line carrier and code tracking [1]. We also found in the published works Bayesian methods using Markov Chain Monte Carlo (MCMC) techniques. The goal of these methods is to estimate time and frequency offset of the GPS signal in the case of interference and jamming protection [14]. In the work of Boutoille and al [10] the code tracking is off line and it fused the data obtained on two different carriers. In this approach the limitations of the method comes from the difficulty to synchronize the data delayed by the ionospheric effect.

The GPS signal discriminator is a measurement that allows to compute the time delay between the CDMA code of the received signal and the local CDMA code generated by the receiver. We deduce from the discriminator value the pseudo range satellite receiver. The new GPS civil signals $L2C$ and $L5$ have a data less pilot channel (The $CL$ code at $f_{L2}$ frequency). We show that in this case we can suppose the discriminator signal piecewise linear. In this context the long integration of the GPS signal consists to segment the signal and then to estimate the parameters of the dynamic model in stationary pieces. The dynamic model assumes a constant acceleration in each segment. Furthermore we propose to fuse the information provided by the data channel in order to improve the SNR. This fusion can be done if we can estimate the message of navigation in the tracking process. We propose to use in this work a state estimate coupled to a change point estimate in order to estimate the code delay. We also propose to use a circular state filter [11] to estimate the phase of the signal and the message of navigation.

The first contribution of this work is the proposition of a new architecture for the tracking of the phase and code delays of a GPS signal. The interest of this new architecture is to fuse the parameters of the signal obtained on the pilot and data channels. The second contribution described in this article is the proposition of a change point estimate of the slope of the discriminator that integrates prior information and fuses the measurements obtained for the pilot and data channels. The article is organised as follow; the first section is dedicated to the description of the GPS model and the problem positioning. The second section describes the code delay estimate and the third section presents the phase tracking process. The experimentation is described in the fourth section. Finally a conclusion ends the paper.

# 2 Problem positioning

## 2.1 GPS Model

The GPS signal, coming from the satellite $s$, consists of a code CDMA $C_s(...)$ modulated at frequency $f_s$. The signal is binary phase shift keying (BPSK) modulated (the L5 signal is Quadra Phase Shift Keying modulated). The code and carrier are assumed to be both time and frequency shifted. A GPS receiver realizes the signal acquisition in a first stage. It estimates the carrier frequency $\hat{f}_s$, the code delay $\hat{\tau}_s$ and the phase delay $\hat{\phi}_s$ of the received signal. At the end of this first stage the receiver tracks variations of these parameters as a function of time in a tracking module. The frequency and phase indeed changes with the Doppler Effect (the relative speed satellite receiver), and the code delay with the satellite receiver pseudo range evolution. In order to compute the navigation solution the receiver multiplies the received signal with in phase and quad phase replica of the GPS signal:

$$\begin{aligned} I_s(t) &= n(t)A_sC_s(t-\tau_s)C_s(t-\hat{\tau}_s) \\ &\quad \cos(2\pi(f_s-\hat{f}_s)t+\phi_s-\hat{\phi}_s)+\eta_s^I(t) \quad (1) \\ Q_s(t) &= n(t)A_sC_s(t-\tau_s)C_s(t-\hat{\tau}_s) \\ &\quad \sin(2\pi(f_s-\hat{f}_s)t+\phi_s-\hat{\phi}_s)+\eta_s^Q(t) \quad (2) \end{aligned}$$

In these expressions $A_s$ is the signal amplitude; it is normalized to drive the noise variance of $\eta_s^I(t_k), \eta_s^Q(t_k)$ to 1. It is a function of the SNR. The noise is supposed to be white, Gaussian, and centred. In this work we consider two measurements of $I(...)$ and two measurements of $Q(...)$ obtained for the pilot channel with $n(t) = 1$ and for the data channel with $n(t) \in \{-1, 1\}$. For $L2C$ these four measurements are obtained for the codes $CL$ and $CM$ for the time multiplexing pilot and data channels. For $L5$ these four measurements are obtained from two carriers of the signal in quadratures. We then define $\tau e = \tau_s - \hat{\tau}_s$, $fe = f_s - \hat{f}_s$, $\phi e = \phi_s - \hat{\phi}_s$ the respective code delay, phase and frequency errors. The acquisition module provides initial values of the GPS signal parameters, so these errors are supposed to be close to zero at the beginning of the tracking stage. The goal of the tracking module is to maintain these errors as small as possible. Parameters are estimated every millisecond, the period $Tc$ of the code CDMA. The GPS signal is down converted (in order to be sampled) at an intermediate frequency by means of a down-converter driven by a local clock oscillator. In this context the clock noise disturbances are modelled as normal random walks [7]. Then we have at the $k^{th}$ millisecond :

$$\begin{aligned} fe_k &= fe_{k-1} + \alpha_{k-1}\, Tc_{k-1} + wf_k \quad (3) \\ \phi e_k &= \phi e_{k-1} + 2\pi\, fe_k\, Tc_k + w\phi_k \quad (4) \end{aligned}$$

In these expressions $wf_k$ and $w\phi_k$ are respectively the noise clock disturbances on the phase and frequency. $\alpha_{k-1}$ models the speed variation of the frequency linked to the movements of the receiver. We suppose here that the accelerations linked to the clock noise are neglected. $Tc_k$, the code length defined at the receiver depends on the carrier frequency, so we have :

$$Tc_k = Tc \;\; \frac{f_s}{\hat{f}_s + fe_k} \quad (5)$$

Finally the error expression of the code delay, which is a function of the relative speed satellite receiver at the $k^{th}$ millisecond, is given by:

$$\tau e_k = \tau e_{k-1} + Tc_k \;\; \frac{f_r - (\hat{f}_s + \tilde{f}e_k)}{f_r} \quad (6)$$

In this expression $\tilde{f}e_k = \sum_{j=1}^{k} \alpha_{j-1}\, Tc_{j-1}$ is the error on the frequency associated to the movement of the receiver and $f_r$ is the intermediate frequency of the emitted satellite signal. Parameters of the received signal are estimated with the values of $I(t)$ and $Q(t)$ integrated on one or several periods $Tc$ of the signal. We define for each satellite $s$ :

$$I_k(\Delta) = n(t)\sqrt{2\ S/N_0\ Tc}\ R(\tau e_k - \Delta) \operatorname{sinc}(fe_k + \frac{\alpha_k\ Tc_k}{2}) \cos(\phi e_k) + \eta_k^I \quad (7)$$

$$Q_k(\Delta) = n(t)\sqrt{2\ S/N_0\ Tc}\ R(\tau e_i - \Delta) \operatorname{sinc}(fe_k + \frac{\alpha_k\ Tc_k}{2}) \sin(\phi e_k) + \eta_k^Q \quad (8)$$

In these expressions $R(..)$ is the normalized function of correlation of the C/A CDMA code. sinc(...) is a function that models the error on the estimate of the carrier frequency. In general the errors values of $\phi e_k$ and $fe_k$ are estimate in a filter with $I_k(0)$ and $Q_k(0)$. The values of $\tau e_k$ are estimated in a filter from the in phase and quad phase observations $Ie_k = I_k(-Tb/2), Il_k = I_k(Tb/2)$ and $Qe_k = Q_k(-Tb/2), Ql_k = Q_k(Tb/2)$ [1]. $Tb$ is the length of one bit of the CDMA code. Finally the value of the RSB is obtained when $I_k(0)$ is maximum, so for $fe_k = 0$, $\tau e_k = 0$ and $\phi e_k = 0$.

## 2.2 Problem Positioning

The coherent discriminator "early-minus-late" is defined by the following expression for which we fix to $Tb$ the code delay between the early and late code generated by the receiver:

$$Z_k = Ie_k - Il_k = n_k\sqrt{2\ S/N_0\ Tc}\operatorname{sinc}(fe_k + \frac{\alpha_k\ Tc_k}{2}) \cos(\phi e_k)\ \ (R(\tau e_k - Tb/2) - R(\tau e_k + Tb/2)) + \eta_k^Z \quad (9)$$

In this expression $\eta_k^Z$ is a white Gaussian noise of variance 2. The expression of the code function of correlation is given by:

$$R(\tau) = \left(1 - \left|\frac{\tau}{Tb}\right|\right) \quad (10)$$

Every millisecond, the discriminator value is processed and the local code is delayed or advanced in order to minimize the delay with the received code. The goal of this process of retiming is to obtain discriminator values close to zero. When the satellite moves away from the receiver, we show that the theoretical variation of the discriminator early-minus-late strictly increases for $|\tau e_i| < \frac{Tb}{2}$. The variation strictly decreases for $\frac{3Tb}{2} > |\tau e_i| > \frac{Tb}{2}$ and stays equal to zero for $|\tau e_i| > \frac{3Tb}{2}$. We display figure 1 the discriminator variations obtained when the code is not retiming in grey and in dark grey the piecewise linear variation of the discriminator when the code is retiming. We show that for $|\tau e_i| \leq Tb/2$ the discriminator variation is proportional to the code delay $\tau e_i$ and is defined by:

$$Z_k = 2\ n_k\sqrt{2\ S/N_0\ Tc}\operatorname{sinc}(fe_k + \frac{\alpha_k\ Tc_k}{2}) \cos(\phi e_k)\ \frac{\tau e_k}{Tb} + \eta_k^Z \quad (11)$$

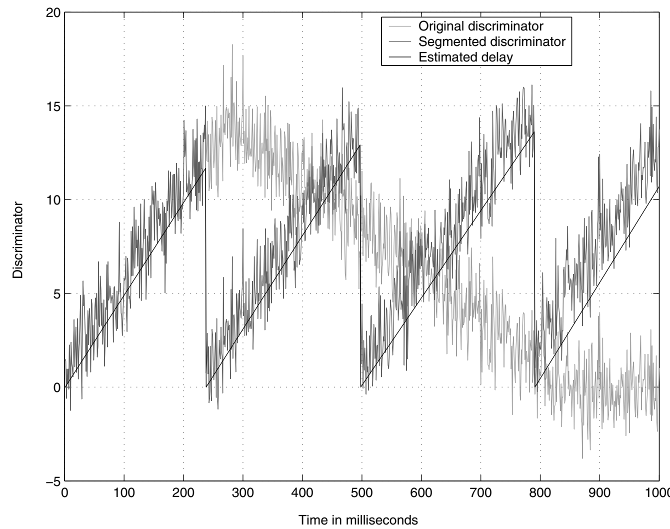


Figure 1: Variation of the retiming discriminator

We suppose here that the receiver trajectory do not modify the strict increasing (the satellite moves away) or decreasing (the satellite comes closer) of the discriminator. We deduce from 11 the expression of the discriminator for $|\tau e_i| < \frac{Tb}{2}$.

In the classical approach the code tracking is realized with a matched filter [2]. In this case the local code is retiming every millisecond with a fix delay. The code is advanced or delayed as a function of the discriminator sign. In our approach, the code is retiming when the discriminator is maximum. At this instant the code is delayed of $Tb/2$ and the pseudo range satellite receiver increases of $C\ Tb/2$, where $C$ is the speed of light.

In order to segment the discriminator (to detect its maximum values) two processes of estimation are working in parallel. The PLL process which estimates the phase and message of navigation of the process. These estimate are used by the DLL process which detects the maximum of the discriminator. When a maximum is detected the codes of the DLL are retiming and the values of the pseudo range between the instant of change and the preceding instants of change are computed. We describe in section 3 the DLL implementation and in section 4 the PLL implementation.

# 3 Recursive estimate of the code delay

## 3.1 Definition of the estimate in the stationary case

The discriminator is described by a random variable $Z_k$ with a Gaussian distribution of variance 2. The variations of the discriminator are described by a piecewise linear model. We consider the following temporal series, defined in each stationary segment by:

$$Z_k = a_k\,(k - t_{i-1}) + \eta_k^Z \qquad t_{i-1} + 1 \leq k \leq t_i \quad (12)$$

$t_i$ is the $i^{th}$ instants of change in the discriminator. $a_k$ is the slope of the discriminator which depends on the speed satellite receiver and of the GPS signal parameters described in equation 11 of the previous section. Let $X_k^{(1)}$, be the mean value of the discriminator at instant $k$. We suppose the process piecewise stationary and we define the discriminator evolution by the following recursive equation:

$$X_k^{(1)} = \sum_{r\in\{t_i\}_{\{i\in N\}}} X_{k-1}^{(1)}(1-\delta_{k-1-r}) + a_k\Delta t + \omega_{k-1}^{(1)} \quad (13)$$

In this expression the discriminator is retiming at instants $r \in \{t_i\}_{\{i\in N\}}$. The Dirac $\delta_i$ impulse takes the value 1 for $i = 0$, else 0.
$\omega_k^{(1)}$ is a white centred Gaussian noise of variance $Q^{(1)}$. Let , $Z^{(1)}$ and $Z^{(2)}$ be the discriminator measurements provided by the pilot and data channels. The equation that links the measurements to the mean value of the process is given by:

$$Z_k^{(1)} = X_k^{(1)} + \eta_k^{Z(1)} \quad (14)$$
$$Z_k^{(2)} = n_k\ X_k^{(1)} + \eta_k^{Z(2)} \quad (15)$$

In this modelling $\eta_k^{Z(\ell)}$ is a white centred Gaussian noise of variance $R^{(\ell)} = 2$, with $\ell \in \{1, 2\}$. In this work we want to estimate the mean value $X_k^{(1)}$ of the piecewise stationary process. The proposed recursive estimate combines a change point estimate with a recursive estimate of the mean value of the process in each stationary segment. In this context the change point estimate provides an estimate of the change instant $\hat{t}_i$. The value of $Q^{(1)}$ defines the confidence on the modelling equation 13, which supposes the linear evolution of the process in the stationary segments. The slope value $\hat{a}_k$ is supplied by the change point estimate that computes the change instant and the discriminator slope after the change. The slope is supposed to be constant in each stationary segment.

The recursive estimator of the discriminator in the stationary peaces is defined in a Bayesian framework. Let $\hat{X}_{k-1}^{(1)}$ and $P_{k-1}^{(1)}$ respectively define the *a posteriori* mean and variance of $X_{k-1}^{(1)}$. The predicted state of the filter is defined by the following expressions that suppose the process stationary and Gaussian:

$$\hat{X}_{k/k-1}^{(1)} = \hat{X}_{k-1}^{(1)} + a_k\ \ \Delta t \quad (16)$$
$$P_{k/k-1}^{(1)} = P_{k-1}^{(1)} + Q^{(1)} \quad (17)$$

In these expressions we suppose that the process has a linear variation and a model accuracy defined by $Q^{(1)}$. Then we have the following recursive estimate of the mean $\hat{X}_k^{(1)}$ and variance $P_k^{(1)}$:

$$P_k^{(1)} = \left(\frac{1}{P_{k/k-1}^{(1)}} + \frac{1}{R^{(1)}} + \frac{1}{R^{(2)}}\right)^{-1} \quad (18)$$

$$\hat{X}_k^{(1)} = P_k^{(1)}\left(\frac{\hat{X}_{k/k-1}^{(1)}}{P_{k/k-1}^{(1)}} + \frac{Z_k^{(1)}}{R^{(1)}} + \frac{n_k\ Z_k^{(2)}}{R^{(2)}}\right) \quad (19)$$

In equations 19 and 18 we suppose independent the error on the discriminator measurements obtained for the pilot and data channel. This assumption is correct for the $L2C$ GPS signal where the codes $CL$ and $CM$, respectively associated to the data and pilot channel, are time multiplexing. Regarding the $L5$ signal the inphase and the quadraphase component, associated respectively to the data and pilot channel, are coded with two different pseudo-random noise (PRN) ranging codes. We suppose in this case that the correlation between these two noisy random components is low and negligible.

### 3.2 Change point estimate

When the discriminator is retiming, its evolution is described by the following recursive equation:

$$X_k^{(1)} = \left(X_{k-1}^{(1)} + \Delta t\left(a_k + \Delta a_r\ H_{(r+1)}(k)\right)\right) \times \left(1 - H_{(1+\frac{3r}{2})}(k)\right) + \omega_{k-1}^{(1)} \quad (20)$$

with $r = t_i$. In this expression $H_{r+1}(k)$ is the Heaviside function that takes he value 1 for $k \geq r+1$ else 0. $\Delta a_r$ is the slope difference at the change instant. Let suppose a change of slope in the model of the process, defined by equation 20, of magnitude $\Delta a_r$. Then we have the expression of the estimate mean $\hat{X}_{k,r}^{(1)}$ at instant $k$ with $k > r$ which is given by:

$$\hat{X}_{k,r}^{(1)} = \hat{X}_{k,0}^{(1)} + \rho_{k,r}(\Delta a_r) \quad (21)$$

In this expression $\hat{X}_{k,0}^{(1)}$ is the estimate mean obtained if we suppose that there is no change in the model. $\rho_{k,r}(\Delta a_r)$ is a function of $\Delta a_r$ that defines the gap with the estimate mean when we consider a change at instant $r$ in the model. We show that $\rho_{k,r}(\Delta a_r)$ is defined by:

$$\rho_{k,r}(\Delta a_r) = \Delta a_r\ \ \rho_{k,r}\ = \Delta a_r \sum_{i=1}^{k-r}\prod_{j=k-i}^{k-1}\frac{P_{j+1}^{(1)}}{P_j^{(1)} + Q^{(1)}} \quad (22)$$

Let $\epsilon_k$ be the difference between the fused measurements and the estimate mean:

$$\epsilon_k = \hat{X}_{k,0}^{(1)} - \frac{\left(R^{(2)}Z_k^{(1)} + n_k\ R^{(1)}Z_k^{(2)}\right)}{R^{(1)} + R^{(2)}} \quad (23)$$
$$P\epsilon_k = P_k^{(1)} + \left(\frac{1}{R^{(1)}} + \frac{1}{R^{(2)}}\right)^{-1} \quad (24)$$

Let define $\underline{\epsilon}$ as a set of innovation values. These values are computed for all the instants defined between $r$ and $k$, with $r < k$. $k$ is the current instant for which the last estimate of the mean value has been computed. The value of $r$ is searched in a working window of size $M$, then between $k - M + 1$ and $k$. The statistical distribution of the innovation is defined by the following expression:

$$P(\underline{\epsilon}, r, \Delta a_r) = P(\Delta a_r/\underline{\epsilon}, r) \quad P(\underline{\epsilon}/r) \quad \pi(r) \tag{25}$$

In this expression $\pi(\underline{r})$ is the prior law on the change instant $r$ and $\Delta a_r$ is a parameter. Furthermore we have:

$$P(\Delta a_r/\underline{\epsilon}, r) \quad P(\underline{\epsilon}/r) = h(\underline{\epsilon}/r, \Delta a_r) \quad P(\underline{s}/r) \tag{26}$$

where $h(..)$ is the likelihood function. Finally the estimate of the change instant is defined by:

$$(\hat{r}) = \underbrace{Argmax}_{(k-M+1 \leq r \leq k)} \quad P\left(r|\underline{\epsilon}\right) \tag{27}$$

Let :

$$(\hat{r}) = \underbrace{Argmin}_{(k-M+1 \leq r \leq k)} \quad -log\left(h\left(\underline{\epsilon}|r; \mu, V\right) \pi\left(r; \sigma_d\right)\right) \tag{28}$$

where $\mu, V$ and $\sigma_d$ are hyper parameters that respectively define prior information on the value of $\Delta a_r$ and the change instant. In our case the likelihood distribution is given by:

$$h(\underline{\epsilon}|r, \Delta a_r; P\epsilon) = \prod_{i=r}^{k} \frac{1}{\sqrt{2\pi P\epsilon_i}} \exp\left(-\frac{(y_i - \Delta a_r \rho_{i,r})^2}{2P\epsilon_i}\right) \tag{29}$$

In practice, we have an inaccurate estimate of $\Delta a_r$ deduced from $a_k$. We use this value as a prior information. We suppose that $\Delta a_r$ has a Gaussian distribution of mean $\mu$ and variance $V/n_r$ [13], with $n_r = k - r + 1$. Then we have:

$$P(\Delta a_r|r; \mu, V) = \frac{\sqrt{2\pi V}}{n_r} \exp\left(-n_r \frac{(\Delta a_r - \mu)^2}{2V}\right) \tag{30}$$

Then we show that :

$$\begin{aligned} h(\underline{\epsilon}|r; \mu, V) &= \left(\frac{1}{4\pi}\right)^{n_r/2} \sqrt{\frac{V_r \ n_r}{V}} \qquad (31) \\ & \exp\left(-\frac{1}{2}\left(\sum_{i=r}^{k} \frac{\epsilon_i^2}{P\epsilon_i} + \frac{n_r \ \mu^2}{V} - \frac{\mu_r^2}{V_r}\right)\right) \end{aligned}$$

with:

$$T_r = \sum_{i=r}^{k} \frac{\rho_{i,r}^2}{P\epsilon_i} \tag{32}$$

$$V_r = 1/\left(T_r + \frac{n_r}{V}\right) \tag{33}$$

$$\mu_r = V_r \left(\mu \frac{n_r}{V} + \sum_{i=r}^{k} \frac{\epsilon_i}{P\epsilon_i} \ \rho_{i,r}\right) \tag{34}$$

The prior probability $\pi(r)$ to have a change instant at $r$ is defined by a Bernoulli law :

$$\pi(r; \sigma_d) = \lambda_r(1 - \lambda_r) \tag{35}$$

where $\lambda_r = 1/2$ for $k - M + 1 \leq r \leq k - \sigma_d$ else $\lambda_r = 0$. This prior law defines the minimum number of samples $\sigma_d$ you must have after the change to detect it. The estimate of $r$ in the working window is given by:

$$\begin{aligned} (\hat{t}_i) &= \underbrace{Argmin}_{(k-M+1 \leq r \leq k)} \{\sum_{i=r}^{k} \frac{\epsilon_i^2}{P\epsilon_i} + \frac{n_r \ \mu^2}{V} \qquad (36) \\ &- \frac{\mu_r^2}{V_r} - \ln\left(n_r \ V_r\right) - \ln \frac{\lambda_r}{(1 - \lambda_r)}\} \end{aligned}$$

The estimate of the magnitude of the change $\Delta a_r$ that maximizes the likelihood distribution 29 is given by:

$$\widehat{\Delta a}_{\hat{t}_i} = \frac{\sum_{i=\hat{t}_i}^{k} \frac{\rho_{i,\hat{t}_i} \ \epsilon_i}{P\epsilon_i}}{\sum_{i=\hat{t}_i}^{k} \frac{\rho_{i,\hat{t}_i}^2}{P\epsilon_i}} \tag{37}$$

In our approach we suppose constant the speed satellite receiver after the change instant, and we have $\Delta a_r = \frac{3}{2} a_k$. Then, finally a change is detected in the $i^{th}$ segment if $\widehat{\Delta a}_{\hat{t}_i} \in [-\frac{3}{2} a_{\hat{t}_i} - \sqrt{V} \ \ -\frac{3}{2} a_{\hat{t}_i} + \sqrt{V}]$. Then we have $\hat{a}_{\hat{t}_i+1} = \frac{2}{3} \widehat{\Delta a}_{\hat{t}_i}$ the slope of the $(i+1)^{th}$ stationary segment that begin at $\hat{t}_i + 1$.

# 4 Recursive estimate of the phase delay

## 4.1 Recursive estimate of the carrier phase delay

The phase delay of the carrier is an angular measurement obtained with the values of $Ip_k$ and $Qp_k$ by the following expression:

$$\phi e_k = \text{Arctag*}\left(\frac{Qp_k}{Ip_k}\right) \tag{38}$$

In this expression Arctag*(...) is the quadrant-specific inverse of the tangent [3]. This measurement is described by a circular random variable $\phi e_k$ with a circular distribution of von Mises of mean $m$ and a parameter of concentration $\kappa$:

$$h_V(\phi e_k) = \tfrac{1}{2\pi I_0(\kappa)} \ e^{\kappa cos(\phi e_k - m)} \tag{39}$$

$I_0(\kappa)$ is the modified Bessel function of order zero.
The phase delay is described by a process $\phi e$ and the following temporal series that model the measurements:

$$\phi e_k = \left(\left[m + \eta_k^{\phi e}\right] \quad mod\left[2\pi\right]\right) \tag{40}$$

$\eta_k^{\phi e}$ is a centred von Mises random variable. Let $X_k^{(2)}$ be the mean phase delay at instant $k$. We define the phase delay evolution by the following recursive state equation:

$$X_k^{(2)} = X_{k-1}^{(2)} + \omega_{k-1}^{(2)} \tag{41}$$

$\omega_k^{(2)}$ is a white centred noise with a von Mises distribution. The parameter of concentration of the distribution is $Q^{(2)}$. Let, $\phi e^{(1)}$ and $\phi e^{(2)}$, be the phase delay measurements obtained respectively from the pilot and data channels of the signal. The equations that link the measurements to the mean values of the processes are given by:

$$\phi e_k^{(1)} = X_k^{(2)} + \eta_k^{\phi e(1)} \tag{42}$$

$$\phi e_k^{(2)} = X_k^{(2)} + \|n_k - 1\| * \frac{pi}{2} + \eta_k^{\phi e(2)} \tag{43}$$

$\eta_k^{\phi e(\ell)}$ is a white centred Gaussian noise with a von Mises distribution. The concentration parameter of the distributions is $K^{(\ell)}$, with $\ell \in \{1, 2\}$. We suppose that the *a posteriori* distribution defined at $k-1$ is a von Mises distribution notice $\mathcal{CN}(...)$. Then we have:

$$X_{k-1}^{(2)} | \phi e_{1:k-1}^{(\ell)} \sim \mathcal{CN}(X_{k-1}^{(2)} | \hat{X}_{k-1}^{(2)}, P_{k-1}^{(2)}) \tag{44}$$

$\hat{X}_{k-1}^{(2)}$ and $P_{k-1}^{(2)}$ respectively define the *a posteriori* mean and variance of $X_{k-1}^{(2)}$. The filter prediction is defined in the stationary cases by the following equation:

$$\hat{X}_{k/k-1}^{(2)} = \hat{X}_{k-1}^{(2)} \tag{45}$$

In our approach, the noise that models the prediction error has a von Mises distribution with a parameter of concentration $Q^{(2)}$. The distribution of the prediction is given by the Chapman-Kolmogorov expression. It is shown in [4] that a von Mises distribution can be approximated with a Wrapped Normal distribution that has the same propriety than in the Gaussian linear case for the sum of random variables. The Wrapped Normal distribution is a sum of Gaussian distributions that allow to find a solution to the Chapman-Kolmogorov expression. Then we can express the variance of the Wrapped Normal distribution [3], noticed $\mathcal{WN}(...)$, as a function of the variance of the von Mises distribution with the function $A(...)$ defined next. Then we have:

$$X_{k-1}^{(2)} | \phi e_{1:k-1}^{(\ell)} \sim \mathcal{WN}\left(X_{k-1}^{(2)} | \hat{X}_{k-1}^{(2)}, A(P_{k-1}^{(2)})\right) \tag{46}$$

With :

$$A(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)} \tag{47}$$

The distribution expression of the prediction is given by :

$$X_k^{(2)} | \phi e_{1:k-1}^{(2)} \sim \mathcal{WN}\left(X_{k-1}^{(2)} | \hat{X}_{k/k-1}^{(2)}, A(P_{k-1}^{(2)}) A(Q^{(2)})\right) \tag{48}$$

This distribution can be approximated by a von Mises distribution as defined previously:

$$X_k^{(2)} | \phi e_{1:k-1}^{(2)} \sim \tag{49}$$
$$\mathcal{CN}(X_{k-1}^{(2)} | \hat{X}_{k/k-1}^{(2)}, A^{-1}\left(A(P_{k-1}^{(2)}) A(Q^{(2)})\right)$$

Then we have the complete equation of the prediction defined by:

$$\hat{X}_{k/k-1}^{(2)} = \hat{X}_{k-1}^{(2)} \tag{50}$$

$$P_{k/k-1}^{(2)} = A^{-1}\left(A(P_{k-1}^{(2)}) A(Q^{(2)})\right) \tag{51}$$

Then we show [8] that the recursive estimate of the mean phase delay and of the parameter of concentration is given by:

$$\hat{X}_k^{(2)} = \arctan(D_k / C_k) \tag{52}$$

$$P_k^{(2)} = \sqrt{C_k^2 + D_k^2} \tag{53}$$

With:

$$C_k = P_{k/k-1}^{(2)} \cos \hat{X}_{k/k-1}^{(2)} + K^{(1)} \cos \phi e_k^{(1)} + K^{(2)} \cos \phi e_k^{(2)} \tag{54}$$

$$D_k = P_{k/k-1}^{(2)} \sin \hat{X}_{k/k-1}^{(2)} + K^{(1)} \sin \phi e_k^{(1)} + K^{(2)} \sin\left(\phi e_k^{(2)} + \|n_k - 1\| * \frac{pi}{2}\right) \tag{55}$$

## 4.2 Detection of the message of navigation

Detection of the message of navigation is realised with the circular distance between the predicted phase delay obtained by equation 51 and the measurements obtained with data channel. Let define the following circular distance:

$$d_k = 1 - \cos\left(\phi e_k^{(2)} - \hat{X}_{k/k-1}^{(2)}\right) \tag{56}$$

We have the following approximation:

$$d_k \approx 1 - \cos\left(\|n_k - 1\| * \frac{pi}{2} + \eta_k^{\phi d}\right) \tag{57}$$

$$P_{\eta_k^{\phi d}} = A^{-1}\left(A(P_{k-1}^{(2)}) A(Q^{(2)}) A(K^{(2)})\right) \tag{58}$$

Where $\eta_k^{\phi d}$ is a white centred von Mises distribution with a parameter of concentration $P_{\eta_k^{\phi d}}$. Then the expression of $d_k$ is given by:

$$d_k \approx 1 - n_k \cos\left(\eta_k^{\phi d}\right) \tag{59}$$

$$\tag{60}$$

The test is defined by:

$$d_k \underset{n_k=1}{\overset{n_k=-1}{\gtrless}} 1 \tag{61}$$

where the distribution and critical values of $\cos(\eta_k^{\phi d})$ are given in [4].

The proposed tracking architecture is composed of two tracking loops which work in parallel. The first loop tracks the code of the pilot and data channel with a noncoherent discriminator. This loop allows to measure the phase delay variations. The values obtained at instant $k$ are fused with the circular filter and the message of navigation is extracted.

The second tracking loop uses at each instant $k$ the data (phase and message of navigation) provided by the first tracking loop. The phases of the pilot and data channels are corrected but not the codes. Then we can measure with this loop the discriminators variations. These measurements are provided at instant $k$ to the change point estimate.

When a change is detected at instant $k$ and at position $\hat{r}$ the code is retiming at instant $k$. The pseudo range values are then calculated for instants between $k$ and the previous change point.

# 5 Experimentation

In this experimentation we want to assess the proposed architecture of fusion. We simulate the GPS signal in a real context. These experiments are realized and described for a static receiver and for a circular dynamic trajectory. Positions are obtained, every millisecond, for each C/A code correlation measurement and the signal is sampled with a 20 MHz frequency. In this experimentation we compare the results obtained with the proposed method and the results obtained with a classical matched filter [2]. We simulate a real GPS constellation the third day of the GPS week 1291 at 12 hours, 00 minute and 00 second. The receiver antenna localization is supposed to be on the roof of the laboratory. We report in table 1 the different parameters of simulation. The algorithm is demonstrated using simulated GPS signals with C/A codes for the pilot and data channels (like in the $L5$ case). The clock noise model parameters are defined for a temperature-compensated crystal oscillator. The values of the phase and frequency random-walk intensities are $S_f = 5 \times 10^{-21} s$ and $S_g = 5.9 \times 10^{-20} s^{-1}$.

Table 1: Simulation parameters

| HDOP | Number of satellites | $C/N_0$ (dB.Hz) | $V/K^{(\ell)}/Q^{(1)}/Q^{(2)}$ Variances |
|---|---|---|---|
| 1.43 | 6 | 50 / 40 | 1/1/0.01/1000 |

We display on figure 2, positions computed every millisecond in the static case, for a signal to noise ratio of $C/N_0 = 50$ dB.Hz . On these figures, we display positions processed with the proposed method in dark and with a standard method in grey. The standard method is the classical match filter [2]. Positions calculated with the standard method are scattered around the real position. In the other hand, with the proposed fusion method the calculated positions in black are closed to the real position.

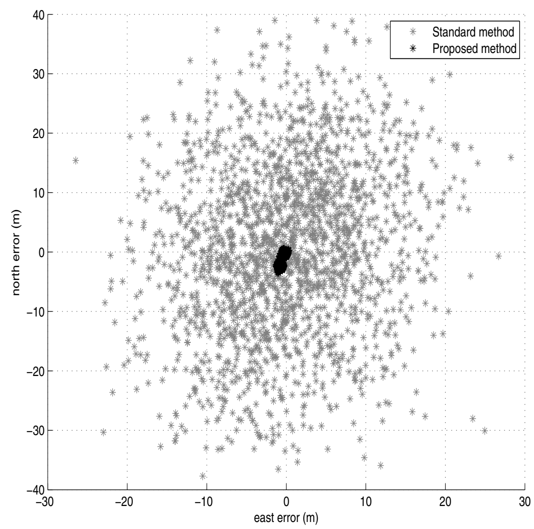


Figure 2: Static positions computed for a $C/N_0$ of 50 dB.Hz

We report on table 2 the mean gap values and variance, according to North and East axis, between the real and estimate positions. We also describe the mean Euclidian distance between the real and estimate receiver positions. Values reports in this table are obtained for different signals to noise ratio with the standard and the proposed fusion method.

Table 2: Error on estimate static positions: **fusion**/standard

| SNR dBHz | Mean gap East dir | Mean gap North dir | Mean Distance |
|---|---|---|---|
| $C/N_0$=50 | **0.5**/6.7 | **1.3**/11.5 | **1.4**/14.6 |
| $C/N_0$=40 | **1.7**/10.7 | **2.7**/19.9 | **3.4**/24.6 |

For the dynamic case, we consider a circular trajectory of radius 10 m cover in 2 s. We show on figure 3, the estimate positions obtained in the dynamic experimental case. We notice for the standard method that points are scattered around the real trajectory like in the static case.

We report on table 3 errors on computing positions in the dynamic case. Compared to the static case, the errors we obtain for the standard method are practically the same. However the errors obtained with the proposed method are smaller than for the standard case but greater than in the static case. Furthermore, we notice like in the static case that the accuracy decreases

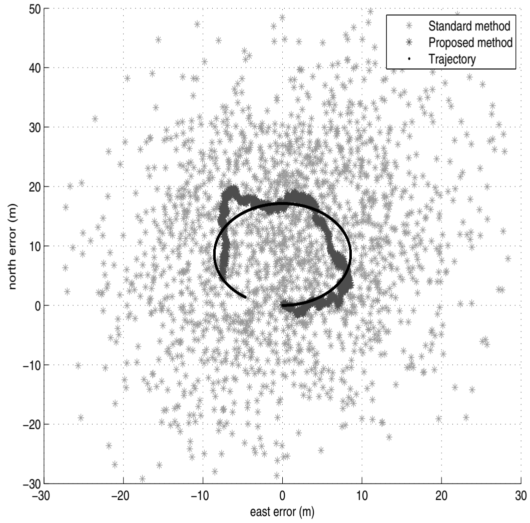


Figure 3: Estimate dynamic trajectory for a $C/N_0$ of 50 dB.Hz

with the SNR.

Table 3: Error on estimate dynamic positions: **fusion**/standard

| SNR dBHz | Mean gap East dir | Mean gap North dir | Mean distance |
|---|---|---|---|
| $C/N_0$=50 | **1.8**/8.4 | **2.9**/14.8 | **3.7**/18.5 |
| $C/N_0$=40 | **2.6**/11.5 | **6.7**/21.8 | **7.7**/26.7 |

# 6 Conclusions

We introduce in this article the architecture of a fusion system for the tracking of the new GNSS signals. The proposed system used a change point estimate of the discriminator values and a circular filter for the phase delays. It fuses the code and phase delays obtained for the pilot and data channels of the GNSS signal. We introduce in this article the recursive implementation of the circular filter and of the change point estimate. These filters fuse the data and use prior information on the parameters of the change detection. The results obtained in the experimentation show that the computed positions obtained with the proposed method are more accurate than the positions obtained with the classical matched filter.

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